REMOTE STORAGE

FREE CONVECTION OF HEAT IN LIQUIDS

BY

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FREE CONVECTION OF HEAT IN LIQUIDS

By Roy A. Nelson

Abstract

Free convection of heat from a hot wire in various liquids.—(1) *Photographic study of the convection streams.* Both a parallel ray and a schlieren method were used and gave similar results. Photographs are reproduced for water, olive oil and glycerine. In all cases there is a necking down of the stream a few millimeters above the wire. When the wire is first heated, a layer of liquid all around the wire is heated; the lower half first moves down, then stops and goes up joining the stream which follows the upper half, which had proceeded as a distinct semi-circular pulse. The appearance of the image of the wire tends to confirm Langmuir's suggestion of a stationary film on the surface. Bubbles on the wire in olive oil were observed to execute interesting oscillations. (2) *Variation of heat loss with the excess temperature of the wire over that of the liquid.* After the wire (a silver one, .33 mm in diameter) had been heated electrically for about 10 minutes a steady state was reached; then the resistance of the wire was measured and the mean temperature of the liquid was determined with a mercury thermometer. It was found in the case of water, alcohol, CCl_4, glycerine and castor oil, that the heat loss per cm^2 per sec. is equal to $b\theta^n$ when n has the values 1.15, 1.105, 1.125, 1.25 and 1.24 respectively. The measurements of A. H. Davis are shown to give values of n of about 1.15 for five liquids, but there is an indication that n depends upon the diameter of the wire. The factor b varies with the liquid and the mean temperature and also perhaps with the diameter of wire.

Part I. Photographic Study

OF THE large amount of work that has been done on convection of heat, practically all has been from the viewpoint of the rate of loss from heated bodies and the effect of the physical properties of the fluid on this rate. Very little has been done on the motions and conditions that exist in the fluid due to convection currents.

Pramanik[1] has made an optical study of free and forced convection from hot wires in air. He gives a number of photographs showing the convection currents, both when the currents are gravity currents and when a stream of air is forced past the wire. He also shows the effect of having several heated wires close together.

In the present work, which was begun before Pramanik's work appeared, a large number of photographs have been taken, showing the convection currents in liquids of widely different physical properties.

[1] Pramanik, Proc. Indian Assoc. Cult. Sci. 7, 115, 1922

1. *Methods of obtaining photographs.* Since the index of refraction of a liquid changes with the density, it is not difficult to make the convection currents in liquids visible and to photograph them.

In this work two methods were used to obtain the photographs; the "parallel light" method, and the "method of striae" or a modification of the Schlieren Method.

The first method is shown in upper part of Fig. 1. An arc light D was placed at the focus of the lens L_1 so that a parallel beam of light passed through the box with the glass sides containing the liquid. The box was 7.5 cm by 8 cm, and 10 cm high. The heating element was a manganin wire 2.7 cm long and 0.0315 cm in diameter. L_2 and L_3 were camera lenses and a magnification of from 3 to 4 diameters was used in obtaining the photographs.

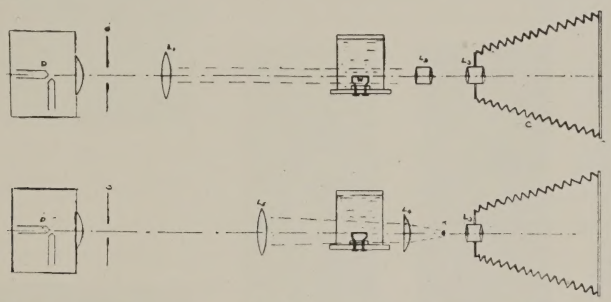
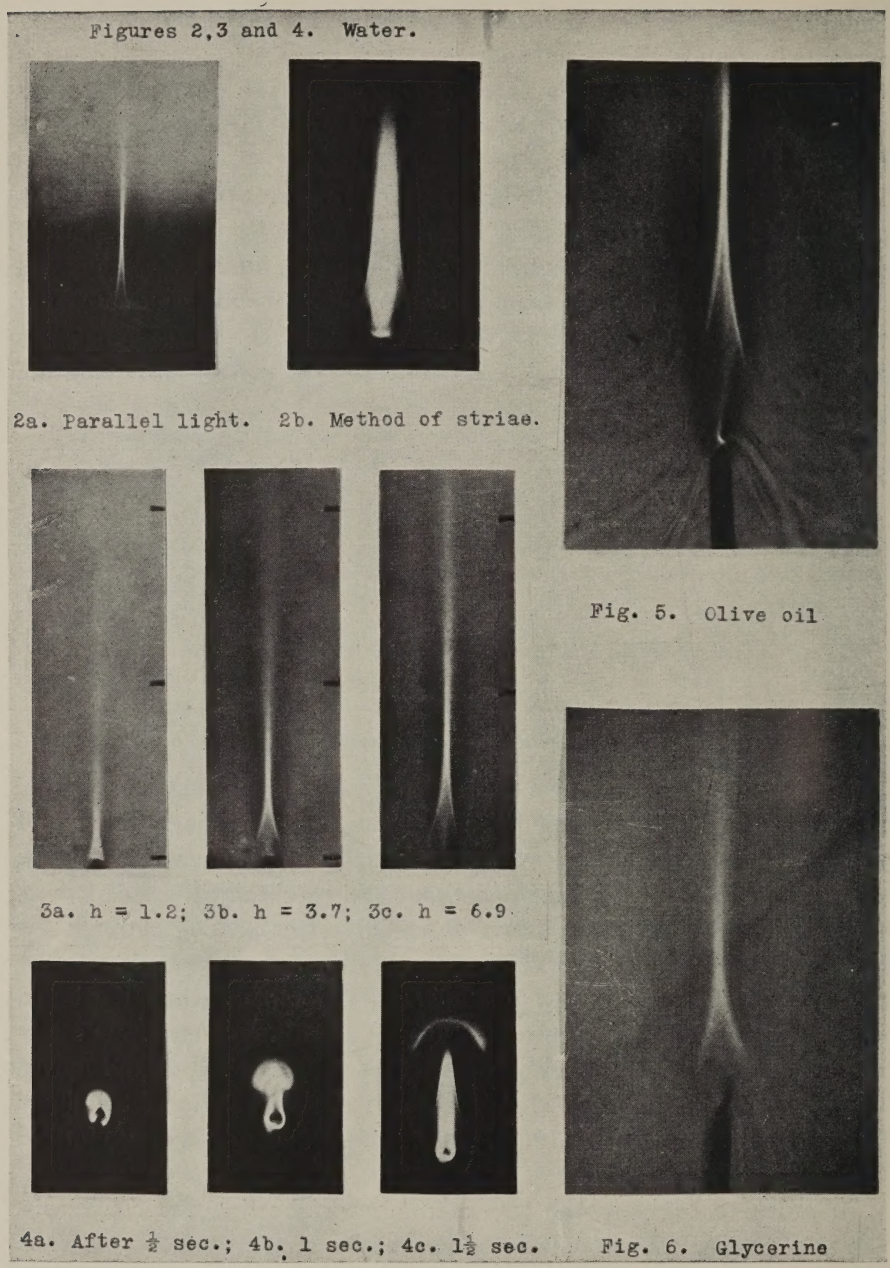


Fig. 1. Optical arrangements; parallel light method above; method of striae below.

In the second method, also shown in Fig. I, a long focus lens L_5 was placed so that a converging beam of light passed through the liquid along the axis of the heating wire. To shorten the distance between the camera and the wire so a magnification could be obtained, a lens L_4 was used to cause the beam to converge rapidly after passing through the liquid. A steel hemisphere K, 0.47 cm in diameter was placed so that practically all the light was shut off from the camera when the liquid was all at the same temperature. When the heating current was established, the liquid in the neighborhood of the wire was heated and moved upward. The difference in the densities of different parts of the liquid caused the light to be bent around the obstacle K and an image was formed on the plate in the camera.

In order to compare the two methods for showing the convection streams, photographs were taken of convection streams under almost the same conditions. The photograph in Fig. 2a was taken by the parallel light method and Fig. 2b by the method of striae, the magnifications being about 5 times and 3.5 times respectively. The same general characteristics may be seen in both photographs. The parallel light method


Figures 2,3 and 4. Water.
2a. Parallel light. 2b. Method of striae.
Fig. 5. Olive oil
3a. h = 1.2; 3b. h = 3.7; 3c. h = 6.9
4a. After ½ sec.; 4b. 1 sec.; 4c. 1½ sec.
Fig. 6. Glycerine

Convection streams in water, olive oil and glycerine
Fig. 2. Comparison of two photographic methods; water
Fig. 3. Variation with heat loss h, in cal. per cm^2 per sec.
Fig. 4. Variation with time after starting the current.

shows the convection streams at a greater distance above the wire because a larger region was illuminated in using this method. Photographs taken using the schlieren method were found to show more details of the convection stream near the wire and so this method was used when the convection flow near the wire was studied. The stream of rising fluid acts as a lens for the beam of light passing through it. This was shown by changing the position of the lens L_2 in Fig. 1. The convection stream could be made to appear bright as shown in the photographs or, if the lens L_2 was moved farther from the box the image would be dark with a bright edge.

If the glass box was turned so that the light passed through the liquid perpendicular to the axis of the wire, it was found that the image of the wire could be made much larger when the heating current was on than when it was off. If the lens L_2 was moved toward the wire a position could be found where the image of the wire practically disappeared when the heating current was turned on. This spreading out of image of the wire seemed to be the same above the wire as it was below. This is what would happen if Langmuir's theory that there is a stationary film on a heated surface is true for liquids. The spreading of the image would not be the same below the wire as it is above if the liquid film was not stationary but was being constantly replaced by unheated liquid. Some of the photographs which will be discussed later on also tend to confirm this theory.

2. *Convection currents in water.* In water it was found that it was necessary with the size wire used to have a rate of heat loss of at least 0.8 calories per cm^2 per sec. in order that convection streams would be visible. Figs. 3a, 3b, and 3c show photographs taken of water having heat losses of 1.28, 3.71, 6.86 cal per cm^2 per sec., respectively. The distances between the horizontal lines of the photographs represent distances of one centimeter. The photographs show that at a short distance above the wire the width of the convection streams is the largest. This is no doubt due to the heated liquid coming out from below the wire. But at a short distance farther up the stream narrows down and is very sharp. Then as it proceeds up it begins to diffuse out into the cooler liquid. It is interesting to note that there is a region in which the flow is practically linear. Figs. 3b and 3c show this quite distinctly.

Figs. 4a, 4b, 4c show the formation of the convection stream. These photographs were taken, using the method of striae. The heat loss in these four cases was all the same, 6.7 cal per cm^2 per sec, but they were taken at $\frac{1}{2}$, 1 and 1.5 sec., respectively, after the current was turned on. Fig. 4a shows the heated layer close to the wire and the distance moved upward is very small. From visual observation, the beginning of the

convection flow had the appearance of a wave moving outward from the wire, but when the lower half had proceeded a short distance down it was reflected and moved upward. Fig. 4b shows this wave moving up. The central dark spot represents the position of the wire. Fig. 4c shows the wave which proceeds upward. The wave could be seen until it reached the surface of the water. These photographs all seem to substantiate Langmuir's film theory. If the molecules of the liquid received heat only as they came in contact or very close to the wire, the heated liquid would not be seen at the lower side of the wire.

If the box was turned so the light passed through the liquid at right angles to the axis of the wire and the current was turned on, the wave shown in the previous photographs appeared as a "wave front" moving upward. Over the central portion of the wire this "wave front" was straight unless there was some disturbing condition such as bubbles or particles clinging to the wire. Above the ends of the wire the end effect could be seen and the "wave front" was seen to curve downward.

3. *Convection currents in alcohol.* Using alcohol as the convecting liquid, the same general characteristics were observed. The stream does not converge as sharply as in the case of water but convection currents for a lower rate of heat loss could be made visible. The velocity of the initial wave which travels up is higher than in water because of the lower viscosity. It was noticed in alcohol and not in the other liquids tried, that near the surface the bright image had a tendency to move slowly back and forth, even when the liquid as a whole seemed perfectly quiet.

4. *Convection currents in olive oil and glycerine.* Fig. 5 shows the stream lines in olive oil after the heat current had been on for fifteen minutes and a steady state had been reached. When the heating current was first turned on a wave of the same nature as shown in Fig. 4 moved slowly upward.

Fig. 6 shows the same effect in glycerine. This photograph was taken 25 minutes after the heating current was turned on.

An interesting phenomenon was observed in olive oil with the light passing through the box perpendicular to the wire. When the wire was heated so that bubbles formed on the wire, these bubbles would move back and forth along the wire. If there was only one bubble it would vibrate over nearly the entire length of the wire. If there was more than one bubble, they would oscillate back and forth, rebounding when two came close to each other. After some time the bubble would come to rest. When the heating current was turned off the bubbles would rise to the surface. This same phenomena was observed in glycerine but the oscillatory motion was damped out much quicker.

Another point of interest in this connection was that if bubbles were formed when the heating current was turned on, and if after the wave like that shown in Fig. 4c had proceeded a centimeter or two above the wire the switch was opened, the bubbles would rise rapidly until they reached this wave and then rise to the surface with it, but would not pass through the wave.

Part II. Measurements of Heat Losses From a Wire

The first experimental study of free convection seems to have been made by Dulong and Petit.[2] They studied the rate of loss of heat from thermometer bulbs in gases maintained at a constant temperature. They found that the rate of loss of heat by convection was proportional to $\theta^{1.233}$ where θ represents the excess in temperature of the heated body over that of the gas.

Lorenz[3] solved mathematically the case for the rate of heat loss from a vertical plate by making certain assumptions regarding the air currents

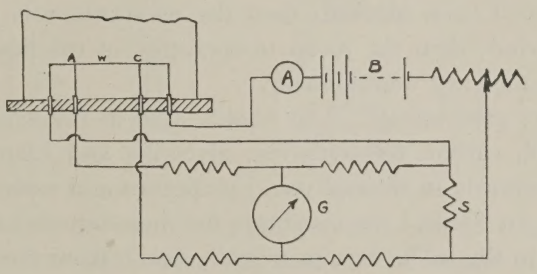


Fig. 7

near the plate. He found that the rate of heat loss should be proportional to $\theta^{5/4}$. This has been verified by experiments on small vertical plates by Langmuir,[4] and on large vertical plates at the National Physical Laboratory of Great Britain.[5] Langmuir[6] has also shown that for heated wires in gases the same exponent of θ holds for temperatures up to 600°C.

The present investigation was undertaken to determine whether the variation of convection with the temperature excess for liquids follows a simple law such as that for gases: $h = b\theta^n$.

1. *Experimental methods.* The apparatus used is shown diagrammatically in Fig. 7. The heating wire was of silver and was 0.033 cm in

[2] Dulong and Petit, Ann. de Chim. et Phys. **7**, 1817

[3] Lorenz, Ann. der Phys. **13**, 582, 1881

[4] Langmuir, Trans. Amer. Electrochem. Soc. **23**, 299, 1913

[5] Report on "Heat Transmission by Convection and Radiation" published by the Food Investigation Board.

[6] Langmuir, Phys. Rev. **34**, 401, 1912

diameter and 16.9 cm in length. The liquid under investigation was contained in a glass box 25 cm by 10 cm by 12 cm. The temperature of the wire was determined by measurements of its resistance, using a Kelvin Double Bridge. The potential drop leads were silver wires 0.015 cm in diameter and soldered to the heating wire 2.2 cm from the ends. The temperature coefficient of the heating wire was found to be 0.0388. The potential drop over the 0.01 ohm resistance S was compared with that of the silver wire. The heating current was taken from a set of storage batteries having a large capacity, the current thus being maintained amply constant.

From preliminary experiments it was found that the temperature of the wire increased rather rapidly when the heating current was first turned on, while after the current had run for about 10 minutes the change in temperature of the wire was proportional to the time and corresponded to the rise in temperature of the liquid as a whole. In taking the readings the heating current was allowed to run until this steady state had been reached; then the resistance was measured and the liquid stirred; then the mean temperature of the liquid was determined with a mercury thermometer.

2. *Results of experiments.* The liquids used in the experiments were water, alcohol, carbon tetrachloride, glycerine and castor oil. These liquids differ greatly in their physical properties and especially in regard to viscosity. In Table I the constants for these liquids are given. The values given in the table were principally taken from the 7th Edition of the Smithsonian Physical Tables and from Landolt-Börnstein "Physikalisch-Chemisch Tabellen."

TABLE I

Physical constants of liquids studied

Liquid	ρ	η	c	$k\ (10)^4$	$\alpha\ (10)^4$
Alcohol	0.792	0.011	0.46	4.3	11.0
Toluene	0.866	0.005	0.34	3.1	10.9
Aniline	1.023	0.045	0.53	4.1	8.5
CCl_4	1.582	0.0097	0.31	2.5	2.66
Water	1.000	0.0101	1.00	14.0	3.0
Olive oil	0.916	0.989	0.43	3.9	3.9
Glycerine	1.26	8.5	0.73	7.0	6.8
Castor oil	0.969	7.91	0.42	4.2	

ρ = density $\quad \eta$ = viscosity $\quad c$ = specific heat per unit vol.
k = thermal expansion $\quad \alpha$ = coefficient of cubical expansion

In Table II the results of a representative set of readings for water are given. The current through the wire and the resistance are given in the first two columns, the heat loss in calories per cm^2 per sec. h, is given in the third column. θ is the temperature difference between the wire and

the liquid, i.e., $\theta = t_W - t_L$. The value of the exponent n in the equation $h = b\theta^n$ was obtained by plotting the logarithms of h and θ and determining the slope of the lines. The ratio h/θ^n is given in the last column.

TABLE II

Water *Mean temperature 24.8°C* $n = 1.147$

I	R	h	t_W	t_L	θ	θ^n	h/θ^n
4.91	.026030	0.1151	26.28	24.36	1.94	2.143	.0537
8.33	.026312	0.3354	29.30	23.34	4.96	6.276	.0535
12.85	.026866	0.8147	35.34	24.46	10.88	15.45	.0528
15.86	.027397	1.266	41.10	24.86	16.24	24.47	.0517
20.88	.028392	2.273	51.97	25.75	26.19	42.33	.0537
26.40	.029946	3.833	68.94	27.97	40.97	70.71	.0542

The results obtained for the different liquids used are collected in Table III.

TABLE III

Liquid	t	n	h/θ^n	Liquid	t	n	h/θ^n
water	10°C	1.156	.0422	CCl_4	18°C	1.131	.0133
water	19	1.165	.0458	CCl_4	22	1.117	.0144
water	24	1.147	.0525	glycerine	19	1.244	.0099
water	25	1.147	.0533	glycerine	27.5	1.247	.0112
water	32	1.141	.0582	castor oil	6.5	1.26	.00442
alcohol	20	1.105	.0212	castor oil	18.0	1.21	.00601
alcohol	20.5	1.105	.0216	castor oil	34.0	1.27	.00589

A. H. Davis[7] gives values for the relation between the rate of heat loss and the temperature difference. He used a Wheatstone bridge method of measuring the resistance of the heated wire and by compensating leads corrected for the end effects. He assumed that the heat loss per unit length was proportional to θ^2 but his table shows that the ratio H/θ^2 was far from constant.

In Table IV the results given by Davis have been recalculated by the present writer and the value of the exponent n determined for each liquid. The heat loss is expressed in calories per cm^2 per sec. instead of calories per cm length per sec. as in his table.

The results show that the values of the exponent n should be between 1 and 1.25 instead of the value $n = 2$ assumed by Davis.

From the present writers' experiments, therefore, and from the present examination of Davis' measurements, it appears that the rate of loss of heat is not proportional to θ^2 but to θ^n where n has a value somewhat greater than unity. For all liquids tried except glycerine and castor oil, the value of n is not far from 1.15. In using liquids of high viscosity it was very difficult to obtain steady readings and in plotting the graph

[7] A. H. Davis, Phil. Mag. **44**, 933, 1922

from the logarithms of h and θ, it was found that the points were more scattered for glycerine and castor oil than for the other liquids. Davis states that his results for glycerine were the least satisfactory. This may account for the difference between the values of n for glycerine obtained from Davis' results and from those of the present writer.

TABLE IV

Values of h/θ^n, where h = heat loss in cal/cm and θ = excess temperature
(Calculated by the present writer from A. H. Davis' measurements)

Liquid	wire diameter	n	1	2	5	10	20	50
Toluene	0.0083	1.111		.0305	.0305	.0308	.0305	.0307
	0.0155	1.154	.0202	.0198	.0194	.0196	.0199	.0204
CCl_4	0.0083	1.127	.0242	.0238	.0241	.0240	.0241	.0240
	0.0155	1.157	.0166	.0168	.0163	.0167	.0166	.0166
Aniline	0.0083	1.173	.0274	.0286	.0277	.0278	.0276	.0285
	0.0155	1.167	.0205	.0201	.0196	.0197	.0202	.0207
Glycerine	0.0083	1.000		.0368	.0360	.0368	.0395	
	0.0155	1.152		.0183	.0172	.0171	.0176	.0183
Olive oil	0.0083	1.146		.0184	.0185	.0183	.0183	.0186
	0.0155	1.173		.0124	.0118	.0120	.0121	.0125

In Tables III and IV the values of the ratio h/θ^n are not constant but vary with the liquid, the mean temperature of the liquid and probably with the size of the heating wire. However, no definite conclusion can be drawn from the change in the ratio h/θ^n with the wire diameter from Table IV, because the mean temperatures of the liquids were not given by Davis. The value of the exponent n is nearly the same for the different liquids and does not change much when the temperature of the liquid is changed. There is an indication from Table IV that the value of n changes with the size of the heating wire, but the accuracy of the measurements is not sufficient to show definitely whether or not this is true. Since n is nearly the same for liquids of widely different physical properties, it is evident that the value of the coefficient b must be the predominating factor in determining the free convection of heat by a liquid.

It is planned to carry out further investigation on the loss of heat by convection in other liquids and determine the effect of the size of heating wire on the value of the coefficient b and on the value of the exponent n. If it is found that the value of the coefficient b is independent of the diameter of the wire, then this coefficient may be taken as a measure of the convective cooling power of a liquid. Even if b is a function of the diameter of the wire, the relative cooling power of different liquids may be found by comparing the values of b when determined with the same size

heating wire. It may also be possible to express b as function of the physical properties of the liquid but so far no simple relationship has been found.

Rayleigh[8] suggests the possibility of applying the principle of similitude to the problem of convection of heat. Richardson[9] and Davis[10] have applied this method and obtained relations for the rate of loss of heat from similar bodies immersed in viscous fluids. Davis has shown that the grouping of variables given by the principle of similitude gives satisfactory results for convection in gas, carbon tetrachloride, aniline, toulene, olive oil and glycerine. He also shows that the thermal conductivity of the liquid should be the predominating physical property in facilitating cooling by convection. The values of the ratio h/θ^n given in Table III bear out this statement, since water has the highest thermal conductivity and also gives the largest value for the ratio h/θ^n.

In a recent paper, Rice[11] has applied Langmuir's film theory and dimensional reasoning to obtain equations for free and forced convection in a liquid. He has used free convection as a method of determining the thermal conductivities of toluene, glycerine, olive oil, aniline, carbon tetrachloride and No. 12 transil oil.

The writer wishes to acknowledge his indebtedness to Professor A. P. Carman for the interest in and the suggestions made in the course of this investigation.

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June, 1923.[12]

[8] Rayleigh, Nature, **45**, 66, 1915
[9] Richardson, Proc. Phys. Soc., May 1920
[10] Davis, Phil. Mag. **40**, 692, 1920; **44**, 329, and 928, 1922
[11] Rice, (Abstract) Phys. Rev. **21**, 474, Apr. 1923
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